efficient; Nu, Nusselt number; Re, Reynolds number; b, jet width; h, height of projection; t, distance between projections; Pr_t, turbulent Prandtl number.

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FORCE ACTION OF A SUPERSONIC DUSTY GAS FLOW ON A BLUFF BODY

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UDC 532.5

Results of a numerical computation of the dusty gas flow around a spherical segment are discussed.

1. Gas deceleration and subsequent energy and momentum redistribution between the particles and the gas occur behind the bow shock in the supersonic flow of a dusty gas around a bluff body. A result is particle retardation and gas acceleration, leading to an increase in the force effect of the gas on the body and a diminution in the action of the particles. The intensity of the exchange depends substantially on the dispersion of the solid phase: For particles of the fine fraction this exchange is realized in a narrow zone adjoining the shock, and in the major part of the perturbed flow the particles and gas are in thermal and kinematic equilibrium; the particles of the coarse fraction are incident on the body surface almost without any change in the kinetic energy they possessed in the free stream. The computation of the force effect is simplified in both limit cases: In the first case, for equilibrium flow behind the shock, it is sufficient to perform the computation of the flow by a pure gas with certain effective parameters dependent on the mass fraction of the solid phase [1]; in the second case summation of the effects of the gas and the particles, determined without taking account of their interaction, is possible. In addition, results of computing the flow of a dusty gas around bluff bodies, obtained taking account of particle deceleration in the shock layer without the reverse influence of the particles on the gas motion ([2] and the bibliography in [3-5]) are presented in the literature. Analysis of a dusty gas flow in a shock layer with interaction between the gas and the particles taken into account is presented in [3-5] without revealing the general regularities of the phenomenon.

Results are discussed below of a numerical computation of the force effect of a dusty gas on a bluff body as a function of the mass fraction and disperseness of the solid phase. Data are presented on the influence of gas and particles separately on an integral force effect on a body, and of the total effect of a dusty gas which would permit estimation of the accuracy of the approximations described above in a specific example.

2. Let us consider the dusty gas flow around a sphere on the basis of a model of a two-velocity and two-temperature continuous medium [6]. Within the framework of this model, a monodisperse cloud of solid particles is considered as a gas deprived of intrinsic pressure.

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Interaction between the gas and particles in this approximation is taken into account by the inclusion of distributed mass forces reflecting the exchange of momentum and energy into the equations of the continuous medium that describe the gas and particle motion.

The results represented in this paper are obtained by solving nonstationary differential equations for the gas and particles, written in divergent form in a spherical (r, θ) coordinate system, by the method of build-up:

$$\frac{\partial \overline{A}_j}{\partial t} + \frac{\partial \overline{F}_j}{\partial \theta} + \frac{\partial \overline{G}_j}{\partial r} + \overline{W}_j + \overline{K}_j = 0, \quad (1)$$

where $\overline{A}_j$, $\overline{F}_j$, $\overline{G}_j$, $\overline{W}_j$ are "vector"-complexes associated with the gas (j = g) and particle (j = s) parameters by the following relationships

$$\overline{A}_j = \{r\rho_j,\ r\rho_j v_{\theta j},\ r\rho_j v_{rj},\ r\rho_j H_j\},$$

$$\overline{F}_j = \{\rho_j v_{\theta j},\ \rho_j v_{\theta j}^2 + p_j,\ \rho_j v_{\theta j} v_{rj},\ \rho_j v_{\theta j} H_j\},$$

$$\overline{G}_j = \{r\rho_j v_{rj},\ r\rho_j v_{\theta j} v_{rj},\ r(\rho_j v_{rj}^2 + p_j),\ r\rho_j v_{rj} H_j\},$$

$$\overline{W}_j = \{\rho_j (v_{\theta j}\mathrm{ctg}\theta + v_{rj}),\ v_{\theta j} W_{1j} + \rho_j v_{\theta j} v_{rj},$$
$$v_{rj} W_{1j} - (\rho_j,\ v_{\theta j}^2 + p_j),\ W_{1j} H_j\},$$

$p_s = 0$ since the cloud of particles is simulated by a continuous medium without intrinsic pressure, and $\overline{K}_j$ is a vector governing the exchange of mass, momentum, and energy between the gas and the particles of spherical shape. Following [1-6] we write for $\overline{K}_j$:

$$\overline{K}_s = \left\{0,\ -r\rho_s\beta(v_{\theta g} - v_{\theta s}),\ -r\rho_s\beta(v_{rg} - v_{rs}),\ -r\rho_s\beta_q\left(\frac{c_s}{c_{pg}}h - H_s\right)\right\},$$

$$\overline{K}_g = \left\{0,\ r\rho_s\beta(v_{\theta g} - v_{\theta s}),\ r\rho_s\beta(v_{rg} - v_{rs}),\ r\rho_s\left(\beta_q\left(\frac{c_s}{c_{pg}}h - H_s\right) + \beta\left(v_{\theta s}(v_{\theta g} - v_{\theta s}) + v_{rs}(v_{rg} - v_{rs})\right)\right)\right\},$$

$$\beta = \frac{3}{4}c_d\frac{\rho_g q}{\rho_s^0 d_k},\quad \beta_q = 6\frac{\mathrm{Nu}}{\mathrm{PrRe}}\frac{\rho_g q}{\rho_s^0 d_k},\quad \mathrm{Re} = \frac{\rho_g q d_k}{\mu}.$$

The following approximation is used in this paper for the dependences of c_d and Nu on Re, Pr and q:

$$c_d = \frac{A(1 + \exp(-0.427/M_q^{4,63}))}{\mathrm{Re}^\delta},\quad \mathrm{Nu} = 2 + 0.6\mathrm{Pr}^{1/3}\mathrm{Re}^{0.5},$$

$A = 24$, $\delta = 1$ for $\mathrm{Re} \leqslant 1$; $A = 24$, $\delta = 0.6$ for $\mathrm{Re} = 1\text{-}10^3$; $A = 0.44$, $\delta = 0$ for $\mathrm{Re} > 10^3$.

Therefore, as in [1-6], the influence of the local perturbations induced in the flow by the other particles on c_d and Nu is not taken into account. Under the conditions of velocity and temperature equilibrium of the free stream, the streaming process is determined by the Mach number M_∞, the adiabatic index γ, and the coefficient of viscosity of the gas and the mass fraction of the particles $\rho_{s\infty}$ in the free stream ($\rho_{s\infty}$ is the quantity of mass of the solid phase per unit volume of mixture referred to the gas density $\rho_{g\infty}$), the size of the particles, and the body.

The solution of the system of equations (1) is sought in the domain bounded by the surfaces, the detached shock, the conical surface $\theta = \theta_k$ under standard boundary conditions [4-5]: the Rankine–Hugoniot conditions for a gas on a shock, the symmetry at θ = 0, the normal velocity vector component is zero on the body surface. Boundary conditions are not posed for the particle parameters on the body surface, therefore particle reflection from the body surface is not taken into account.

A numerical solution of the problem formulated is obtained by using the explicit MacCormack finite-difference scheme of second-order accuracy [7]. The flow around a sphere for $M_\infty = 3$; $\gamma = 1.4$; $\beta = \beta_q = 0$ was considered as a control example. In this case the gasdynamic field

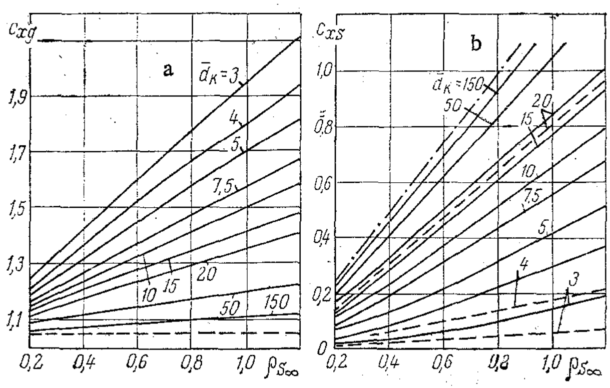

Fig. 1. Drag coefficient component c_x due to the effect of the gas (a) and the particles (b) on a spherical segment in a dusty gas flow (the numbers marking the lines correspond to 0.1 the diameter of the solid particles in μm).

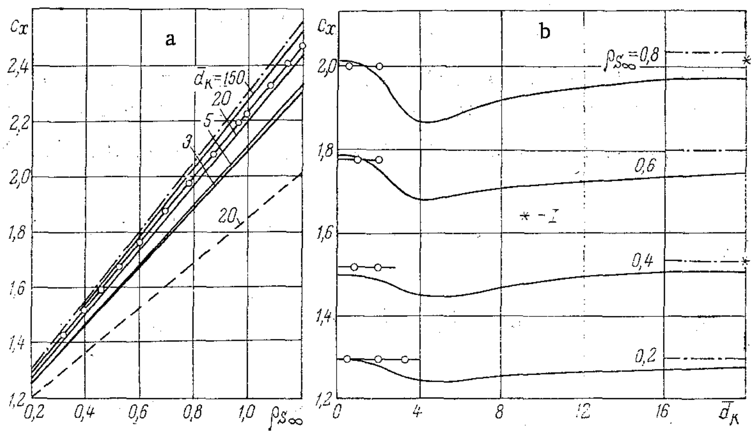

Fig. 2. Dependence of the drag coefficient of a spherical segment on the mass fraction of the solid phase (a) and the particle diameter (b). I refers to 1500 μm ($\bar{d}_k$ = 0.1 d_k μm).

parameters obtained agreed with a 1-2% error with those presented in [8]; the particle parameters differed by an order of magnitude of 0.1% of the corresponding free stream parameters.

3. The flow is considered around a sphere of radius R = 0.27 m by a gas with particles of spherical shape whose diameter varies within 5-1500 μm limits while the mass fraction of the solid phase varies between 0.2 and 1.2. The computations were performed for the following parameters $M_\infty = 3$; $\gamma = 1.4$; $Pr = 0.72$; $\rho_{g\infty} = 0.13$ kg/m³; $\rho_s^0 = 2 \cdot 10^3$ kg/m³; $\mu_\infty = 1.6 \cdot 10^{-4}$ kg/m·sec. The gas viscosity in the shock layer varied in proportion to the square root of the temperature.

The computation results are represented in Figs. 1 and 2. Values of the frontal drag coefficients c_{xg}, c_{xs}, c_x due to the action of the gas, the particles, and the total action of the dusty gas on a spherical segment with angle $\theta_k = 60°$ are presented in these figures. It was assumed in the computation of c_{xs} that the force effect of the particles on the body surface element is related to the particle loss of the momentum component normal to the body surface. Results obtained with the interaction between the gas and particles (solid lines) taken into account, without taking account of particle influence on the gasdynamic field (dashed lines), without taking account of the interaction between the gas and the particles (dash-dot lines), and the results of a computation in the equilibrium approximation (lines with circles) are shown. In the latter case, the computation was performed by the program intended for a computation for a pure gas flow. The effective parameters of this gas (the free stream Mach number and the adiabatic index) are determined by the relations [1]

$$M_{ef}=M_\infty\left\{\frac{(1+\rho_{s\infty})\left(1+\rho_{s\infty}\dfrac{\gamma c_s}{c_{pg}}\right)}{1+\rho_{s\infty}\dfrac{c_s}{c_{pg}}}\right\}^{1/2},\quad \gamma_{ef}=\frac{1+\rho_{s\infty}\dfrac{c_s}{c_{pg}}}{1+\rho_{s\infty}\dfrac{\gamma c_s}{c_{pg}}}\gamma.$$

It follows from Fig. 1a that as the particle diameter increases the force effect of the gas on the body diminishes and tends to the action of a pure gas. In its turn, the force effect of the particles increases as their diameter grows and it approaches the action of particles, in magnitude, determined without taking their deceleration in the shock layer into account (Fig. 1b). The difference between the particle force action on a body determined with gas and particle interaction taken into account and without taking account of particle influence on the gas decreases as the particle diameter grows, and for particles of diameter d_k = 200 μm does not exceed 5% in the example considered. At the same time, the total action of a gas dusted by 200-μm-diameter particles on a spherical segment is determined with an error exceeding 25-30% without taking account of particle influence on the gas. As the particle size diminishes, this error increases. Diminution in the particle size causes an increase in the force effect of the gas and diminution of the particle action. This results in a nonmonotonic dependence of the force effect of the dusty gas on the particle size. The greatest values accrue to c_x for particles of the fine fraction in the build-up of an equilibrium regime in the shock layer and for large size particles when interaction between the gas and the particles is not substantial. It is characteristic that the results obtained in the equilibrium approximation (lines with circles) and without simplification (solid lines) agree with a 1% error (Fig. 2b). This is an illustration of the accuracy of the computations. The minimum on the curves reflecting the change in the frontal drag of a spherical segment that holds in the example considered is related, for d_k = 30 μm, to the more rapid diminution in the force effect of the gas as compared with the increase in the action of the particles caused by the growth of d_k (Figs. 1 and 2b).

As the mass fraction of the solid phase $\rho_{s\infty}$ increases, the force effect of the gas, the particles, and the total action increase almost linearly. A noticeable difference from the linear dependence is observed in the flow around a body by a gas with particles of the fine fraction for c_{xg} and c_{xs} determined with interaction between the gas and particles taken into account.

It follows from Fig. 2b that the difference between the maximum and minimum of the force effect of a dusty gas on a spherical segment does not exceed 10% in the example considered, as the particle size changes in a large range. It hence follows that in approximate computations it is completely admissible to use results obtained under the assumption of a total absence of interaction between the gas and the particles in determining the frontal drag of a body in a dusty gas. In this case, the relationship $c_x = c_{xg0} + \rho_{s\infty}c_{xs0}$, where c_{xg0} is the body wave drag in a pure gas, and c_{xs0} is the body wave drag coefficient determined in a Newton approximation. More accurate estimates can be obtained from computations based on the equilibrium approximation.

NOTATION

v_{θ_j}, v_{r_j}, velocity vector components in a spherical coordinate system; ρ_j, quantity of mass per unit volume; H_g, total gas enthalpy; H_s, specific particle enthalpy; p_g, pressure in the gas; c_s, c_{pg}, specific heats of the particle and gas material for constant volume; d_k, particle diameter; ρ_s^o, density of the particle material; h, specific enthalpy of the gas; $q = \sqrt{(v_{\theta g} - v_{\theta s})^2 + (v_{rg} - v_{rs})^2}$; M_q, velocity and Mach number in particle relative motion in the gas; c_d, particle aerodynamic drag coefficient; Nu, Re, Pr, Nusselt, Reynolds, and Prandtl numbers; M_∞, free stream Mach number; γ, adiabatic index for the gas; ρ_s, quantity of solid phase mass per unit volume of mixture, referred to the free-stream gas density; μ, gasdynamic viscosity coefficient; c_x, frontal drag coefficient in the dusty gas; c_{xg}, component of the sphere frontal drag coefficient in a dusty gas due to the action of the gas; c_{xs}, sphere frontal drag coefficient component in the dusty gas due to the particle action. Subscripts: ∞, free-stream parameters; g, s, gas and particle parameters, respectively, and j = g, s.

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HEAT OR MASS TRANSPORT TO POORLY STREAMLINED BODIES

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The heat and mass transfer between a stream and a streamlined body is examined taking into account the formation of turbulent fluid-filled zones behind the body.

As the stream velocity increases, the unseparated laminar flow around a body is replaced by separation, a vortex zone forms near the root domain of the body in which the mixing intensity increases as the Reynolds number grows. If the Re is not very large, the impurity heat or mass transfer to the body is determined entirely by the process of convective heat conductivity or diffusion, where the magnitude of the local flow on the surface decreases rapidly with distance from the inflow point. In this case the main transfer is realized in the frontal domain of the body and the root domain does not take part in the transfer in practice. As the Re increases, and as turbulence intensifies within the vortex zone, the role of this latter grows considerably and can become dominant for sufficiently large Re [1, 2].

The total heat for mass flux to the body is evidently comprised of flows in the laminar boundary layer domain up to its separation from the body surface and in the turbulized domain after separation. The transfer mechanisms differ substantially in the domains mentioned, and the construction of appropriate models for each requires the involvement of methods of different kinds. The transfer to the body frontal domain reached by the laminar boundary layer, and to the root domain adjoining the turbulized fluid is considered separately below. For definiteness, we shall speak about the stationary diffusion of impurity mass at constant concentrations far from the body and at its surface, but all the results to be obtained will be valid for heat transfer also.

Transfer to the Frontal Domain

The diffusion flow near the surface around which the laminar boundary layer flows can be found from the solution of the convective diffusion equation. If $Pe \gg 1$ was assumed, the known method of a thin diffusion boundary layer [1] is naturally used for the approximate solution. For $Sc \gg 1$ this layer is built-in into the hydrodynamic, for $Sc \ll 1$ on the other hand, the hydrodynamic boundary layer is built-in into the diffusion layer. The fluid velocity within the diffusion layer limits is evidently described by perfectly different relation-

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